Ch. 9: Scaffolding in Mathematical Modeling for ELLs

Cynthia Oropesa Anhalt
The University of Arizona

The Common Core State Standards in Mathematics for English Language Learners: Grades K-8
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***Teachers of English to Speakers of Other Languages* (TESOL)**

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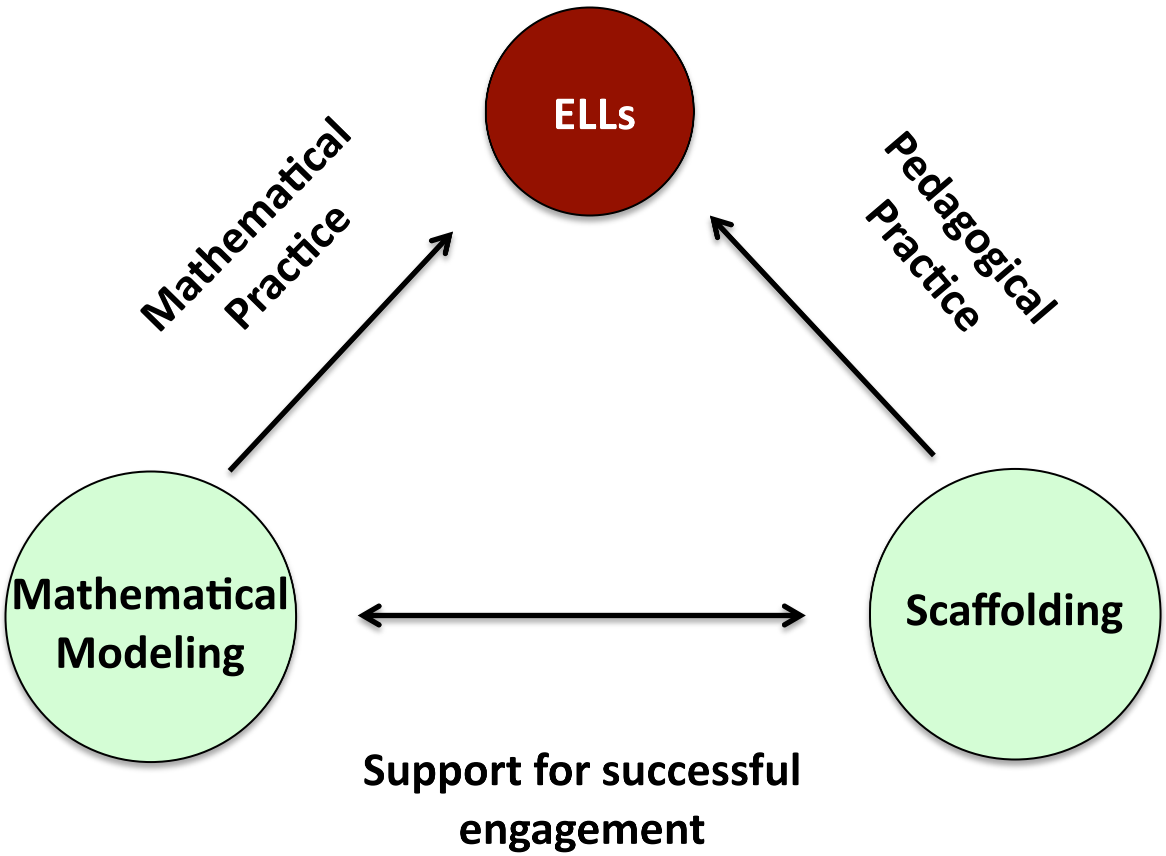
Scaffolding in Mathematical Modeling for ELLs



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Mathematical Modeling (MM)…

- … is a process for addressing problems in everyday contexts or problems within mathematics for which we may not have complete information.
  - may require research, decision making, assumptions
- … problems require students to make productive symbolic descriptions of meaningful situations (Lesh & Harel, 2003).
- … requires students to "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (CCSSM, 2010, p. 7).

Model with Mathematics (MP4) (CCSSM, 2010. p. 7)

Mathematically proficient students can:

- *Apply what they know, make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.*
- *Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.*
- *Analyze those relationships mathematically to draw conclusions.*
- *Routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.*
- *These characteristics of MM imply a cycle.*

Traditional Word Problems v. Mathematical Modeling Problems

Traditional Word Problems	Modeling Problems
Tend to be closed problems	Tend to be open-ended problems
Context is often contrived	Real-life context less contrived
Solutions have defined algorithms	No particular algorithm is defined
All parameters given	Some parameters given, some are assumed
Structure is given	Less structure is given
Assumptions are usually not necessary to solve the problem	Assumptions are part of the process for solving the problem
Usually there is an exact solution	Approximate solutions are acceptable

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Scaffolding as a Pedagogical Practice

- The term ***scaffolding*** refers to the support that teachers provide on elements of a task that are initially beyond a student's capability, thus allowing students to focus on coordinating their existing knowledge with new understandings (Wood, Bruner, & Ross, 1976).

- ***Scaffolding*** as a pedagogical practice is the support given during the learning process, which is tailored to the needs of the students to help them achieve their learning goals (Sawyer, 2006).

Scaffolding Strategies to Support ELLs

Social scaffolding	***Analytic scaffolding***
Collaborative environment in which students work to learn from each other through discourse, such as the offering of ideas, explaining their thinking, and making sense of the problem in structured groups.	Strategies that focus on the meaning of the mathematics, and connections between representations and within mathematical concepts
Collaborative structured teams, purposeful groupings Opportunities to think individually-have pair discussion-share out whole group Communication norms Explanations, justifications, question asking, “wait time”, rephrasing, use of gestures, first language as a resource, public sharing of solutions (Baxter & Williams 2010)	Explicit mathematical connections Demonstrations, Experimentation, data collection, research, modeling process, making assumptions Sustained focus on mathematical meaning, checking for reasonableness of solutions Multiple representations: graphs, diagrams, tables, charts, notation, manipulatives, organizers, reference charts for terminology

Sample Mathematical Modeling Task:
The Car Wash Problem ***(NCTM, 2005)***

A locally-owned automated car wash advertises that it serves millions of satisfied customers each year. Is this a reasonable claim?

Explain and justify your solution.

How would you approach the problem?

CCSSM: 4.OA - Operations and Algebraic Thinking

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity.

From Theory to Practice: The Car Wash Problem

Launching the Problem	Show pictures/video Ask questions to elicit background knowledge about car washes (experience with car washes, how a car wash works, time, etc.)
Posing the problem	Have a student read the problem aloud to class. "Think-Pair-Share" – Making meaning, understanding the problem. Teams figure out what assumptions are needed
Solving the Problem in Collaborative Teams	Discuss, agree, and list relevant assumptions and decisions, determine which numbers can be used to form equations and do computations. Share thinking in teams. Write explanations. All members contribute. Reflect on reasonableness of solution. Create a team poster of solution.
Sharing the Solutions	Students share all components of the problem, including process and justification of solutions.
Class Discussion	After presentations of solutions are made, compare and contrast the various solutions. Encourage students to ask mathematically relevant questions after the team presentations (the teacher may need to model questions initially).

Closure

- The non-routine, open-ended nature of modeling problems are opportunities to let ELL students explore with mathematics what they know to deepen and expand that content knowledge while furthering their English language development

- The teacher's role includes facilitating well-planned strategic social and analytic scaffolding.

- Peer interaction is powerful in productive environments that hold students accountable.

- Actively engaging ALL students, especially ELL students, in mathematical modeling encourages self-reliance and confidence in decision making, problem solving, and communicating ideas.

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Cynthia Oropesa Anhalt
canhalt@math.arizona.edu
The University of Arizona

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Beyond writing and speaking: Broadening mathematical communication with English Language Learners

Anthony Fernandes
University of North Carolina Charlotte
Marta Civil
University of North Carolina
Leslie H. Kahn
Tucson Unified School District

CEMELA

Center for the Mathematics Education of Latinos/as

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Mathematical Practices

- *Construct viable arguments and critique the reasoning of others* (MP #3)
- *Attend to precision* (MP #6)
- *Look for and make use of structure* (MP #7)

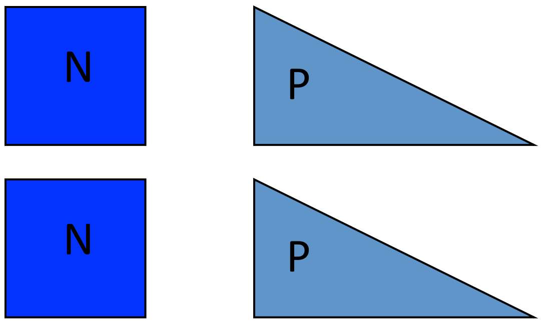
Multimodal perspective

- Multiple modes are involved in communication and representation
  - speech and writing are just two of many
- Modes offer various affordances
  - Speech
  - Drawings
- People choose among the various modes to make meaning
- Meaning is distributed across the modes

Broadening communication

- Pedagogical practice - B*roadening mathematical communication*
- Students' speech and written work often dominate the classroom mathematical discourse
- Teachers interpret communication that includes modes like gestures, manipulation of shapes and drawings

The Area Comparison Problem



Bob, Carmen, and Tyler were comparing the areas of N and P. They each conclude the following:

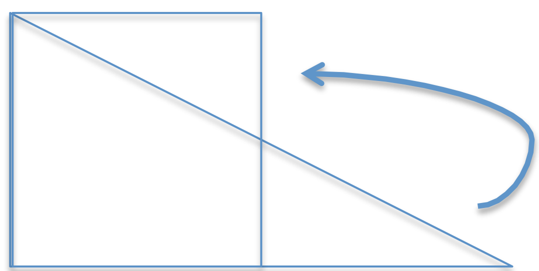
Bob: N and P have the same area.
Carmen: The area of N is larger.
Tyler: The area of P is larger.

Who is correct? Use pictures and words to explain why.

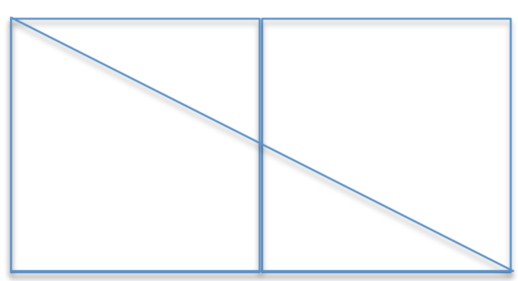
National Assessment of Educational Progress (1996)

Solutions

- Solution 1

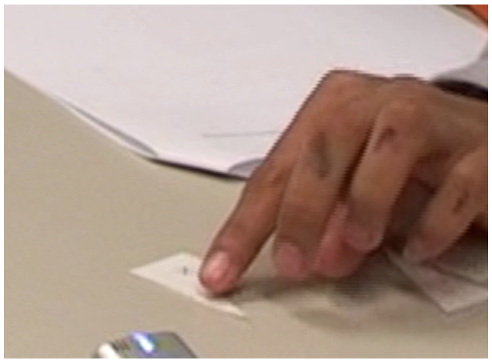


- Solution 2

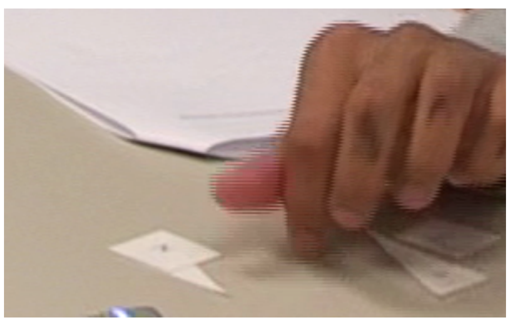


2N =2P , so N=P

Maicon

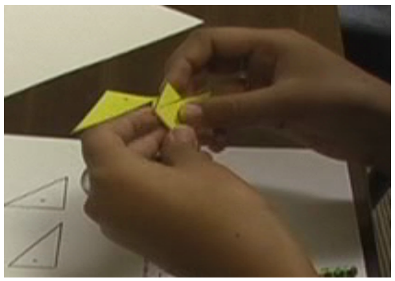


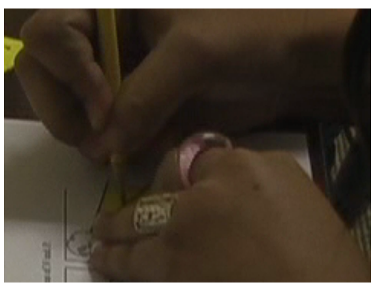
If you cut this out

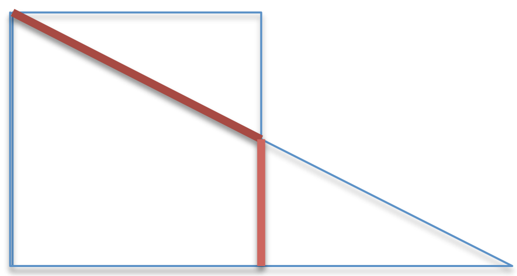


and put it like to make this square

Yolanda





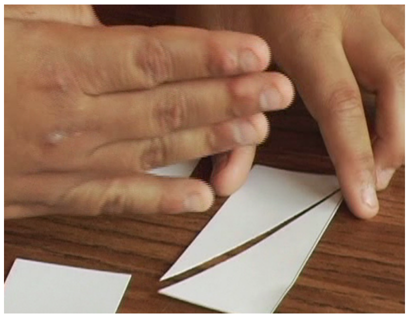


They have the same area

Question

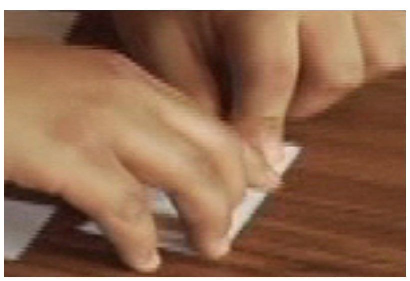
How can you use the idea of multimodal communication in your teaching of mathematics?

Adam



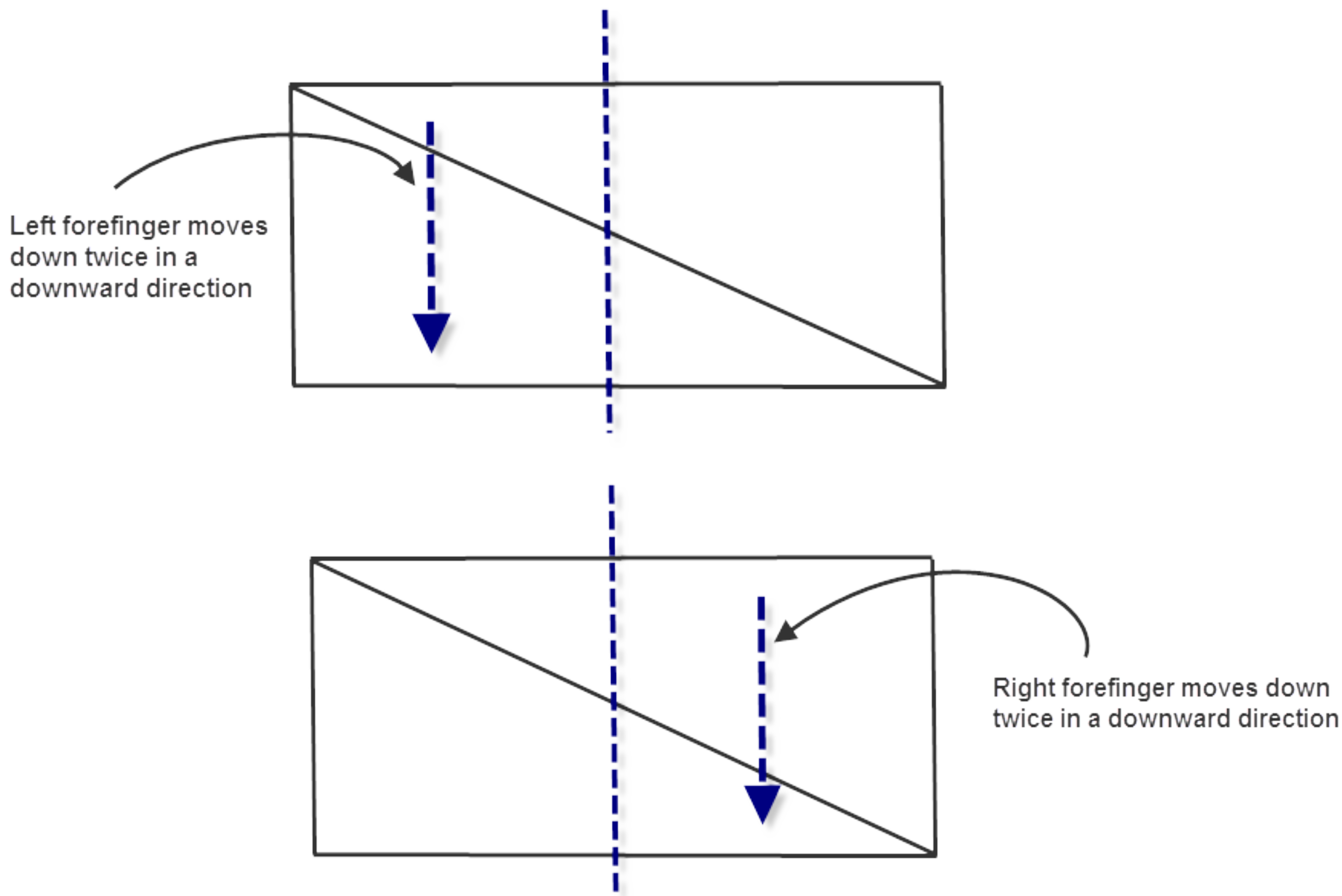
Like say you cut ‘em down the middle straight

right here will be one square



It’ll be one square

Adam



Left forefinger moves down twice in a downward direction
Right forefinger moves down twice in a downward direction

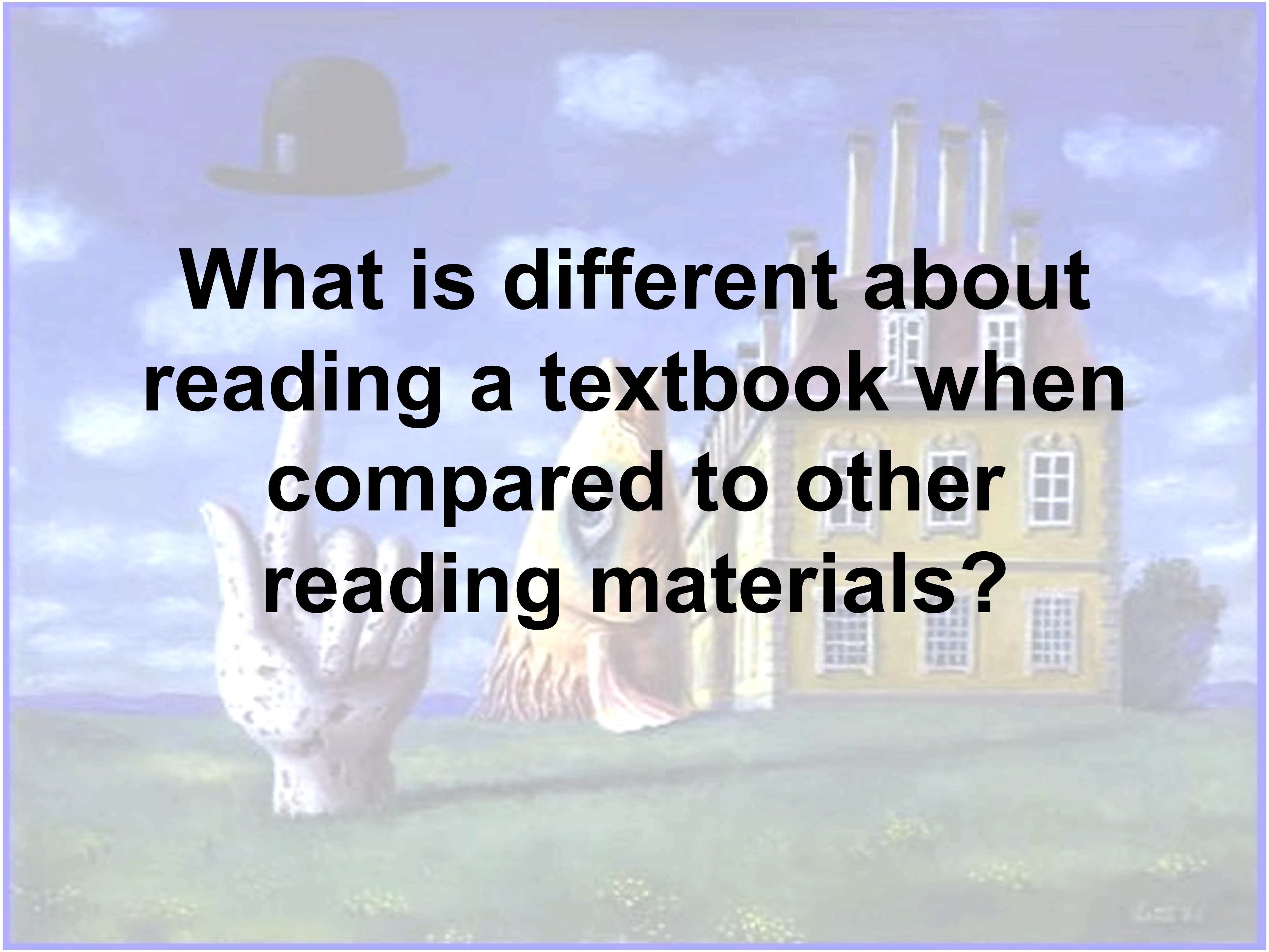
Fostering Mathematical Discourse with Socratic Seminar

Angela Thompson
Governors State University
athompson7@govst.edu

June 27, 2014

Do you ask your students to read a textbook (non-fiction)?

- When?
- Where?
- Why?
- How/how much?
- How often?



What is different about reading a textbook when compared to other reading materials?

Do you *teach* students to read a textbook (non-fiction)?

- Why?
- How/how much?
- How often?

What is a Socratic Seminar?

A method to try to understand information by creating an in-class dialogue based on a specific text. Participants seek deeper understanding of complex ideas through rigorously thoughtful dialogue, rather than memorizing bits of information or meeting arbitrary demands for 'coverage.'

Some Benefits and Skill Gains:

- Critical reading skills
- Conflict resolution
- Respect for diversity
- Positive learning env.
- Community of inquiry
- Collaborative culture
- Engaging
- Cheap
- Authentic
- Wonder

- Self Esteem
- Problem solving
- Listening, speaking, reading, writing
- Validates languages
- Academic language
- Practice wait time
- Helps ALL students
- Flexible
- Metacognitive

Benefits for English Learners:

- Less stressful to speak in class after preparing at home
- Less stressful because no expectation of one right answer
- Autonomy to choose one’s own contributions
- Extra time available to make a response
- Opportunity to incorporate home language
- Peer modeling of reading comprehension
- Promotes equity and empowerment

Summary:

1. Select a text, copy, distribute
2. Students prepare text as homework
3. Assign inner/outer circle; write an opening question; arrange desks
4. Present question; wait 5 minutes
5. Inner circle has a discussion *while* outer circle listens and takes notes
6. Outer circle evaluates the discussion
7. Teacher gives feedback to all

Table 1: CCSS Mathematical Practice Standards in Socratic Seminar

3. Construct viable arguments and critique the reasoning of others.
Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
Make conjectures and build a logical progression of statements to explore the truth of their conjectures.
Analyze situations by breaking them into cases
Recognize and use counterexamples.
Justify their conclusions, communicate them to others, and respond to the arguments of others.
Reason inductively about data, making plausible arguments that take into account the context
Compare the effectiveness of plausible arguments
Distinguish correct logic or reasoning from that which is flawed
Listen or read the arguments of others, decide whether they make sense, ask useful questions
6. Attend to precision.
Try to communicate precisely to others.
Use clear definitions in discussion with others and in their own reasoning.
State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.

Table 2: CCSS Literacy Standards in Socratic Seminar

CCSS English Language Arts Standards for Science & Technical Subjects Grade 6-8
CCSS.ELA-Literacy.RST.6-8.1 Cite specific textual evidence to support analysis of science and technical texts.
CCSS.ELA-Literacy.RST.6-8.2 Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.
CCSS.ELA-Literacy.RST.6-8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6–8 texts and topics.*
CCSS.ELA-Literacy.RST.6-8.6 Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.
CCSS.ELA-Literacy.RST.6-8.8 Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.
CCSS.ELA-Literacy.SL.6.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.
CCSS.ELA-Literacy.SL.6.1a Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion.
CCSS.ELA-Literacy.SL.6.1b Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed.
CCSS.ELA-Literacy.SL.6.1c Pose and respond to specific questions with elaboration and detail by making comments that contribute to the topic, text, or issue under discussion.
CCSS.ELA-Literacy.SL.6.1d Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing.
CCSS.ELA-Literacy.SL.6.3 Delineate a speaker's argument and specific claims, distinguishing claims that are supported by reasons and evidence from claims that are not.
CCSS.ELA-Literacy.SL.6.4 Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.
CCSS.ELA-Literacy.SL.6.6 Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate.

See Chapter 7 for complete details!:

Angela Thompson

athompson7@govst.edu

Integrating Communication in Common Core Mathematics with ELLs

Craig Willey
Indiana University, IUPUI

Bill Zahner
San Diego State University

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Two Studies

#1

- **Context:** multilingual seventh grade class in large, urban elementary school
- **Students:** Largely bilingual Latin@s, about 80% non-native English speakers
- **Teacher:** Ms. Lenihan, a monolingual teacher in her 4th year
- **Curriculum:** developed her own curriculum; some Project-based units; drew heavily from internet resources

#2

- **Context:** bilingual ninth grade algebra class in agricultural community in California
- **Students:** Bilingual Latin@s, about 1/3 enrolled in ELD
- **Teacher:** Ms. Velazquez, an experienced bilingual teacher who used
- **Curriculum:** IMP units supplemented with other handouts

Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

MP3: Construct viable arguments

Mathematically proficient students… **reason inductively about data, making plausible arguments that take into account the context from which the data arose,**

MP3: Construct viable arguments

Mathematically proficient students… **reason inductively about data, making plausible arguments that take into account the context from which the data arose, justify their conclusions, communicate them to others, and respond to the arguments of others.**

What does this practice look & sound like in bilingual classrooms?

MP6: Attend to Precision

- Mathematically proficient students **try to communicate precisely to others**…. [they] express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, **students give carefully formulated explanations to each other.** By the time they reach high school they have learned to **examine claims and make explicit use of definitions.**

What does this precision look & sound like in bilingual classrooms?

Focus of Our Analysis

Two "High-Leverage" Practices:

- Carefully selecting tasks that lead to opportunities to use academic language, and
- Anticipating the language demands of tasks and seizing opportunities to build on *students'* contributions of academic language

Practice #1:

Carefully selecting tasks that lead to opportunities to use academic language

- Is this task problem-based?
- Is this task flexible (i.e., open-ended, lacks explicit parameters)?
- Is this task group-worthy (Cohen, 1996; Cohen & Lotan, 1997; Lotan 2003)?
- Does this task demand social interaction?
- Does this task draw on kids' resources (e.g., intuition, curiosity, lived experiences) or support their ability to "read the world" (Gutstein, 2005)?

Practice #2:

Anticipating the language demands of tasks and seizing opportunities to build on *students'* contributions of academic language

- How do I think structure my class for student-student interactions?
- What are the critical (essential) questions to be posed from this task?
- What supports might I need to put forth to support children's discursive interactions?
- Am I grouping my students strategically?
- What will I listen for?
- Who will I invite to contribute?
- What are the differentiated opportunities to contribute (with differentiated risk-values associated)?
- How will I respond to an under-developed contribution?

How Many Cigarettes Were Produced?

Back To Article

DIRECTIONS: ***Use the data below to make a line graph that shows U.S. cigarette production from 1925 to 1995.***

YEAR	NUMBER OF CIGARETTES	YEAR	NUMBER OF CIGARETTES
1925	80,000,000,000 (80 billion)	1965	529,000,000,000
1935	134,000,000,000	1975	607,000,000,000
1945	340,000,000,000	1985	594,000,000,000
1955	396,000,000,000	1995	487,000,000,000

Why is it important to know what points lie between our data points?

L: Why didn't you put this point, um, like, here (*pointing to a specific point on the graph*)?

O: Because, um, the number's like rounded, almost rounded to 500. You just go a little down.

L: OK. Does that make sense? (*turning to Ramon and Salvador*) You know, 487 is about here, maybe a little less. What's smack dab in the middle of 450 and 500, like if I wanted to put a point right in the middle?

R: (*responding quickly, stuttering*) Seven, seven, five hundred, seven fifty.

L: (*looking at Ramon*) Seven hundred fifty…?

R: Billion.

L: So, 750 billion is in between 450 and 500?

R: (*standing up to get better access to the paper*) Uh, uh, no, it's six hundred fifty billion.

O: Four hundred fifty-five.

S: Four seventy five (*standing up*).

R: Four seventy five.

L: Four seventy five. How do you guys know that there is twenty-five, twenty-five billion from here to here (*pointing to the adjacent interval markers*)?

(*three second pause*)

O: What?

L: How do you know to add 25 billion?

R: Because, like, uh…

(*four second pause*)

O: Ummm…

L: How'd you know that it was right in between?

O: Because 450, and we're counting by 50, so half of 50 is 25.

L: Very good. (*Stands up*) So, half of that is…(*voice trails off as she walks away*)

R: Yeah! (*raising his hand to "high five" Omar*)

Doing Some Great Things

- Task is relevant (the project was generated collaboratively with students) and contextualized
- She routinely prods for explanations: "How do you know?" or, "Why isn't it here?"
- Asks students to evaluate for reasonableness
- Works with students on fundamental skills (i.e., number sense, plotting) in the context of a rich problem-task

Anticipating Language Demands and Seizing Opportunities

- Facilitate student-student discourse
- Consider for whom we are mediating learning
- Ask follow-up questions even after acceptable explanation is given
- Return to central/anchor question (which has implications for maximizing "rigor" and relevance)
- Seize opportunities to extend problems to help kids "read their world" (e.g., When was the pinnacle of cigarette production? What caused the initial and continued decline?)

Thank You

cjwilley@iupui.edu